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Utilizing Problem Structure in Optimization of Radiation Therapy

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Abstract:

In this thesis, optimization approaches for intensity-modulated radiation therapy are developed and evaluated with focus on numerical efficiency and treatment delivery aspects. The first two papers deal with strategies for solving fluence map optimization problems efficiently while avoiding solutions with jagged fluence profiles. The last two papers concern optimization of step-and-shoot parameters with emphasis on generating treatment plans that can be delivered efficiently and accurately.

In the first paper, the problem dimension of a fluence map optimization problem is reduced through a spectral decomposition of the Hessian of the objective function. The weights of the eigenvectors corresponding to the p largest eigenvalues are introduced as optimization variables, and the impact on the solution of varying p is studied. Including only a few eigenvector weights results in faster initial decrease of the objective value, but with an inferior solution, compared to optimization of the bixel weights. An approach combining eigenvector weights and bixel weights produces improved solutions, but at the expense of the pre-computational time for the spectral decomposition.

So-called iterative regularization is performed on fluence map optimization problems in the second paper. The idea is to find regular solutions by utilizing an optimization method that is able to find near-optimal solutions with non-jagged fluence profiles in few iterations. The suitability of a quasi-Newton sequential quadratic programming method is demonstrated by comparing the treatment quality of deliverable step-and-shoot plans, generated through leaf sequencing with a fixed number of segments, for different number of bixel-weight iterations. A conclusion is that over-optimization of the fluence map optimization problem prior to leaf sequencing should be avoided.

An approach for dynamically generating multileaf collimator segments using a column generation approach combined with optimization of segment shapes and weights is presented in the third paper. Numerical results demonstrate that the adjustment of leaf positions improves the plan quality and that satisfactory treatment plans are found with few segments. The method provides a tool for exploring the trade-off between plan quality and treatment complexity by generating a sequence of deliverable plans of increasing quality.

The final paper is devoted to understanding the ability of the column generation approach in the third paper to find near-optimal solutions with very few columns compared to the problem dimension. The impact of different restrictions on the generated columns is studied, both in terms of numerical behaviour and convergence properties. A bound on the two-norm of the columns results in the conjugate-gradient method. Numerical results indicate that the appealing properties of the conjugate-gradient method on ill-conditioned problems are inherited in the column generation approach of the third paper.

Keywords: Optimization, intensity-modulated radiation therapy, conjugate-gradient method, step-and-shoot delivery, column generation, quasi-Newton method, regularization, sequential quadratic programming

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