

# DYNAMIC PHASE BOUNDARY ESTIMATION USING ELECTRICAL IMPEDANCE TOMOGRAPHY<sup>♣</sup>

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## **Abstract:**

In electrical impedance tomography (EIT), an image of the conductivity or permittivity of a domain is inferred from the surface electrical measurements. Typically, conducting electrodes are attached to the periphery of the domain and small alternating currents are applied to some or all of the electrodes. The resulting electrical potentials are measured, and the process is repeated for numerous configurations of applied currents. Image reconstruction in EIT is a kind of nonlinear optimization problem in which the solution is obtained iteratively through forward and inverse solvers. The physical relationship between the internal conductivity profile and surface voltages is governed by a partial differential equation with an appropriate boundary condition. It is impossible to obtain an analytical solution for the forward problem in most cases and a numerical technique such as finite element method (FEM) is often employed. Reconstruction algorithms for EIT fall into two categories. Firstly, the so-called *static imaging* techniques are used for those cases where the internal conductivity of the body is time invariant within the time taken to acquire a full set of measurement data. These static imaging techniques often fail when there are fast impedance changes in the region of interest. In the other category, we have *dynamic imaging* techniques that enhance the temporal resolution for those situations where the conductivity distribution inside the body changes rapidly.

In this thesis, a special class of EIT inverse problems is discussed, in which the position and shape of the objects within the domain need to be identified whereas the conductivities of these objects are known *a priori*. In this class, there are further two types of problems for binary mixtures according to the topology of the boundary to be estimated: open boundary problems in which the object domain can be divided into two disjoint regions which are separated by an open boundary; and closed boundary problems in which the anomalies are enclosed by the background substance. The FEM solution of forward problem using complete electrode model (CEM) is discussed for 2D geometry. After the derivation of FEM solution, the forward solver is modified as a set of coefficients representing the boundary. Two different region boundary representations are used: representation of closed boundary

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<sup>♣</sup> An electronic copy of this thesis and supplementary material is available on author's website

with truncated Fourier coefficients; and representation of open boundary with discrete front points. The analytical Jacobian matrices are derived for both the cases. The Kalman-type reconstruction algorithms are then applied as inverse solvers to estimate the dynamic changes in phase boundaries. The Kalman filter is an efficient recursive filter that estimates the state of a dynamic system from a series of incomplete and noisy measurements. Kalman filter is an important topic in control theory and is modeled on a Markov chain, traditionally built on linear operators and is perturbed by Gaussian noise. Here, the state of the system is represented as a vector of real numbers that model the phase boundary (Fourier coefficients or front points). In order to use the Kalman filter when given a sequence of noisy observations (voltage data) to estimate the internal state of a process, one must model the process in accordance with the framework of the Kalman filter. Therefore, an evolution model of boundary coefficients is used along with an observation model which uses the modified FEM solution of forward problem. To date, different types of Kalman filter (which was essentially based on a linear assumption) appeared in literature, some of them being nonlinear versions of the original Kalman filter (proposed in the sixties). They adopt nonlinearity (either in the process model or in the observation model or in both) to cater to a wide range of nonlinear problems as most nontrivial systems are inherently nonlinear.

Initially, extended Kalman filter is employed to recover front points that represent the interfacial boundary in stratified flows of two immiscible liquids. The results are shown with varying measurement noise levels, front points and contrast ratio. Additionally, an analysis of current injection protocol is given which is helpful in limiting the number of current patterns used to obtain the measurement data. The interacting multiple model technique is introduced as an inverse algorithm for the recovery of front points. It consists of a bank of extended Kalman filters, each working on a different process noise model. Further on, different kinematic models for extended Kalman filter are discussed. These kinematic models are constructed using first- and second-order Markov models. Four different kinematic models are considered to estimate the shape of the elliptic air bubbles in the conducting medium. The unscented Kalman filter is then suggested as an improvement over extended Kalman filter. Since extended Kalman filter uses the linearized version of forward solver so these linearization errors may result in an inaccurate estimation of boundary coefficients when impedance changes abruptly. The unscented Kalman filter is based on the unscented transform, a method that propagates mean and covariance information through a nonlinear transformation, thus precluding the need to use the Jacobian matrix. Experimental validation of unscented Kalman filter is provided by considering a cylindrical phantom with plastic rods (having an infinite resistance) immersed in a conducting medium (saline water). Finally, the Gauss-Newton measurement update in unscented Kalman filter is employed, which improves the performance further due to the iterative nature of the measurement update. Hence, our research concludes that Kalman-type filters offer an efficient real-time solution for the visualization of abrupt changes in phase boundaries.

### **Thesis related journal publications:**

- [1] Nonstationary phase boundary estimation in electrical impedance tomography using unscented Kalman filter  
U. Z. Ijaz, A. K. Khambampati, J. S. Lee, S. Kim, K. Y. Kim  
*Journal of Computational Physics*, 227(15):7089–7112, July 2008  
DOI: 10.1016/j.jcp.2007.12.025

- [2] Electrical resistance imaging of time-varying interface in stratified flows using unscented Kalman filter  
U. Z. Ijaz, S. I. Chung, A. K. Khambampati, K. Y. Kim, S. Kim  
*Measurement Science and Technology*, 19(6) 065501 (11pp), June 2008  
DOI: 10.1088/0957-0233/19/6/065501
- [3] Moving interfacial boundary estimation in stratified flows of two immiscible liquids using electrical resistance tomography  
S. Kim, U. Z. Ijaz, A. K. Khambampati, K. Y. Kim, M. C. Kim, S. I. Chung  
*Measurement Science and Technology*, 18(5):1257-1269, May 2007  
DOI: 10.1088/0957-0233/18/5/012
- [4] Nonstationary phase boundary estimation in electrical impedance tomography based on the interacting multiple model scheme  
B. S. Kim, U. Z. Ijaz, J. H. Kim, M. C. Kim, S. Kim, K. Y. Kim  
*Measurement Science and Technology*, 18(1):62-70, January 2007  
DOI: 10.1088/0957-0233/18/1/008
- [5] Kinematic models for non-stationary elliptic region boundary in electrical impedance tomography  
U. Z. Ijaz, K. Y. Kim  
*Journal of Research Institute of Advanced Technology Cheju National University*, 17(2):25-32, December 2006.